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Fourth Semester B.E. Degree Examination, June/July 2014

Field Theory

Time: 3 hrs.

Max. Marks: 100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

1. a. A charge Q is divided into two parts Q_1 and $Q_2 = Q - Q_1$. At what value of Q_1 , will the electrostatic repulsion force between these two charges be the greatest, at a constant distance between them? (04 Marks)
- b. A charge distribution generates a radial electric field,

$$\vec{E} = \frac{a}{r^2} e^{-\frac{r}{b}} \vec{a}_r$$
 where a and b are constants. Determine the total charge giving rise to this electric field. (04 Marks)
- c. A circular ring of radius 'a' carries a uniform charge ρ_l C/m and is placed on the xy plane with axis the same as the z-axis.
 - i) Show that $\vec{E}(0,0,h) = \frac{\rho_l a h}{2 \epsilon_0 [h^2 + a^2]^{3/2}} \vec{a}_z$.
 - ii) What values of h gives the maximum value of \vec{E} ?
 - iii) If the total charge on the ring is Q , find \vec{E} as $a \rightarrow 0$. (12 Marks)
2. a. A particle is moving under the influence of a force given by $F = -Kx$, where K is constant and x is the distance moved. Then the energy gained by the particle (in Joules) in moving from $x = 0$ to $x = 4$ mtr is nK . Determine the value of n . (04 Marks)
- b. Show that the relaxation time is very less for conductors and very large for dielectrics. (06 Marks)
- c. If $\vec{D} = 10e^{-2z}(\rho \vec{a}_\rho + \vec{a}_z)$ C/m², determine the flux of \vec{D} out of the entire surface of the cylinder $\rho = 1$, $0 \leq z \leq 1$. Confirm the result by using divergence theorem. (10 Marks)
3. a. Derive Poisson's and Laplace's equation in Cartesian coordinates. (04 Marks)
- b. Semi-infinite conducting planes at $\phi = 0$ and $\phi = \frac{\pi}{6}$ are separated by an infinitesimal insulating gap. If $V(\phi = 0) = 0$ and $V(\phi = \frac{\pi}{6}) = 100V$, calculate V and \vec{E} in the region between the plates. (06 Marks)
- c. The bottom surface of a thundercloud of area A and the earth can be modeled as a pair of infinite parallel plates with equal and opposite surface uniform charge densities. Suppose the vertical electric field at the surface of the earth has a magnitude $|\vec{E}_{atm}|$ and points towards the thundercloud,
 - i) Find the expression for the total charge density ρ_s on the bottom surface of the thundercloud. Is this charge density positive or negative?
 - ii) Suppose that the water in the thundercloud forms water droplets of radius 'r' that carry all the charge of the thundercloud. The drops fall to the ground and make a height 'h' of rainfall directly under the thundercloud. Find an expression for the charge on each droplet of water.

Q.No.3(c) continued...

iii) For the drops in part (ii), find an expression for the electric field $|\vec{E}_{\text{drop}}|$ on the surface of the drop due only to the charge on the drop.

iv) If a typical drop has radius $r = 0.5 \text{ mm}$ and the rainfall makes a height $h = 2.5 \times 10^{-3} \text{ m}$,

what is the ratio $\frac{|\vec{E}_{\text{drop}}|}{|\vec{E}_{\text{atm}}|}$?

(10 Marks)

- 4 a. State and explain Amper's law. (04 Marks)
- b. The region $2.5 < \rho < 3.5$, $0 \leq \phi \leq 2\pi$, $0 < z < 2 \text{ cm}$, is current free and contains the field $\vec{H}_{\phi} = \frac{30}{\rho} \text{ A/m}$. Find scalar magnetic potential V_m at $\rho = 2.8 \text{ cm}$, $\phi = 110^\circ$, $z = 0.5 \text{ cm}$, if $V_m = 0$ at $\phi = 0$ and there is a barrier at $\phi = 180^\circ$. (06 Marks)
- c. If $\vec{A} = \rho \cos \phi \vec{a}_\rho + \sin \phi \vec{a}_\phi$, evaluate $\oint \vec{A} \cdot d\vec{\ell}$ around the path shown in Fig.Q4(c).

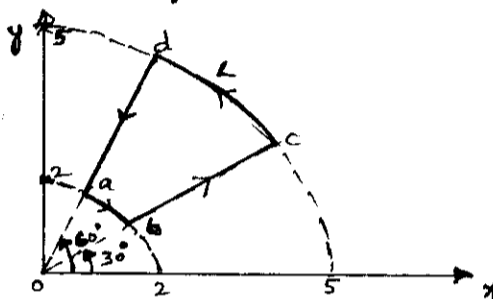


Fig.Q4(c)

(10 Marks)

PART - B

- 5 a. A horizontal metal wire is carrying an electric current from North to South, using uniform magnetic field, it is to be prevented from falling under gravity. So, what should be the direction of this magnetic field? Justify the answer. (04 Marks)
- b. A charged particle moves with a uniform velocity $4\vec{a}_x \text{ m/sec}$ in a region where $\vec{E} = 20\vec{a}_y \text{ V/m}$ and $\vec{B} = B_0\vec{a}_z \text{ Wb/m}^2$. Determine B_0 such that the velocity of the particle remains constant. (06 Marks)
- c. Write the equations to compare electric circuit and magnetic circuit with respect to
- emf and mmf
 - Electric and magnetic scalar potential
 - Electric current and magnetic flux
 - Resistance and reluctance
 - $\oint \vec{E} \cdot d\vec{\ell}$ and $\oint \vec{H} \cdot d\vec{\ell}$

With the help of hysteresis loop, discuss the practical problems that arise in magnetic circuit when compared to electric circuit. (10 Marks)

- 6 a. Write Maxwell's equations for time varying fields in integral form. (04 Marks)
- b. Given $\vec{E} = (x + 2y + az)\vec{a}_x + (bx - 3y - z)\vec{a}_y + (4x + cy + 2z)\vec{a}_z$. Find a, b and c if \vec{E} is irrotational. (06 Marks)
- c. Given the magnetic flux density $\vec{B} = 2 \times 10^{-4} \cos(10^6 t) \sin(0.01x) \vec{a}_z \text{ Tesla}$. Find :
- The magnetic flux passing through the surface $z = 0$, $0 \leq x \leq 20 \text{ m}$, $0 < y < 3$ at $t = 1 \mu\text{sec}$.
 - $\oint \vec{E} \cdot d\vec{\ell}$ around the perimeter of the surface specified above at $t = 1 \mu\text{sec}$, using Maxwell's equation. (10 Marks)

- 7 a. Prove Poynting theorem using Maxwell's equations and hence define Poynting vector. (10 Marks)
- b. A lossy dielectric has an intrinsic impedance of $200 \angle 30^\circ \Omega$ at a particular radian frequency ω . If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component $\vec{H} = 10e^{-\alpha x} \cos\left(\omega t - \frac{1}{2}x\right) \vec{a}_y$ A/m. Find the electric field component \vec{E} , attenuation constant α and skin depth δ . (10 Marks)
- 8 a. Define loss tangent. How do you classify the medium as good conductor or good dielectric based on loss tangent? (04 Marks)
- b. Discuss the following: (06 Marks)
- Standing wave
 - Standing wave ratio
- c. In free space ($z \leq 0$), a plane wave with $\vec{H}_i = 10 \cos(10^8 t - \beta z) \vec{a}_x$ mA/m is incident normally on a lossless medium ($\epsilon = 2\epsilon_0$, $\mu = 8\mu_0$) in region $z \geq 0$. Determine \vec{E}_i , the reflected wave \vec{H}_r , \vec{E}_r and the transmitted wave \vec{H}_t , \vec{E}_t . (10 Marks)
